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# Sequential decision making under uncertainty

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Lisbon, Portugal

Reading group meeting, January 4, 2007





This meeting:

- Overview of the field
  - ▶ Motivation
  - ▶ Assumptions
  - ▶ Models
  - ▶ Methods
- What topics shall we address?
- Fix a schedule.





- Major goal of Artificial Intelligence: build intelligent agents.
- Russell and Norvig (2003): “an agent is anything that can be viewed as perceiving its environment through sensors and acting upon that environment through actuators”.
- Problem: how to act?
- Example: a robot performing an assigned task.





## Reinforcement learning applications:

- Aibo gait optimization (Kohl and Stone, 2004a,b; Sagar et al., 2006)
- Helicopter control (Bagnell and Schneider, 2001; Ng et al., 2004)
- Airhockey (Bentivegna et al., 2002)
- More on

<http://neuromancer.eecs.umich.edu/cgi-bin/twiki/view/Main/>





# Sequential decision making under uncertainty

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Assumptions:

**Sequential decisions:** problems are formulated as a sequence of “independent” decisions;

**Markovian environment:** the state at time  $t$  depends only on the events at time  $t - 1$ ;

**Evaluative feedback:** use of a reinforcement signal as performance measure (reinforcement learning);





# Sequential decision making under uncertainty (1)

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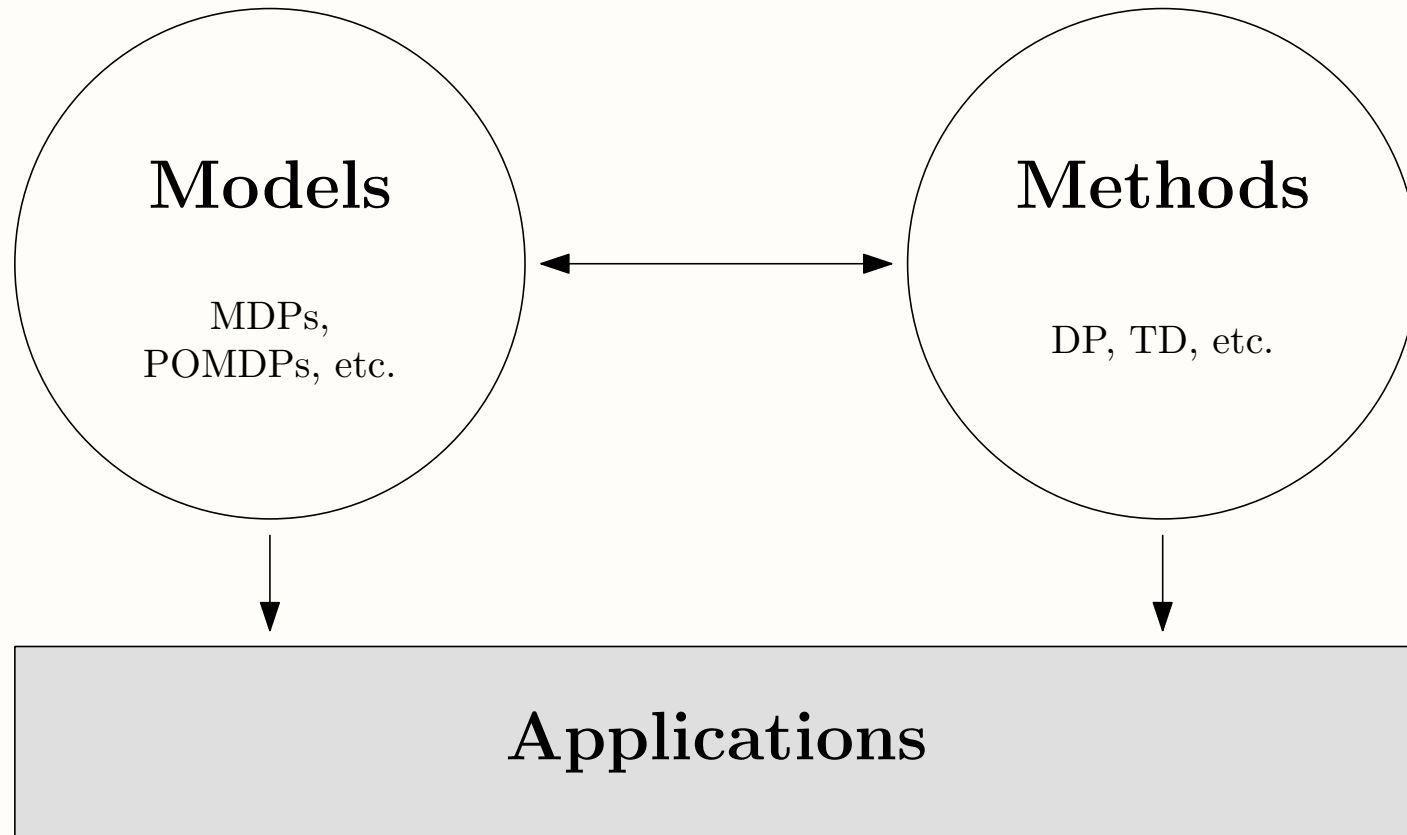
Possible variations:

- Type of uncertainty.
- Full vs. partial state observability.
- Single vs. multiple decision-makers.
- Model-based vs. model-free methods.
- Finite vs. infinite state space.
- Discrete vs. continuous time.
- Finite vs. infinite horizon.



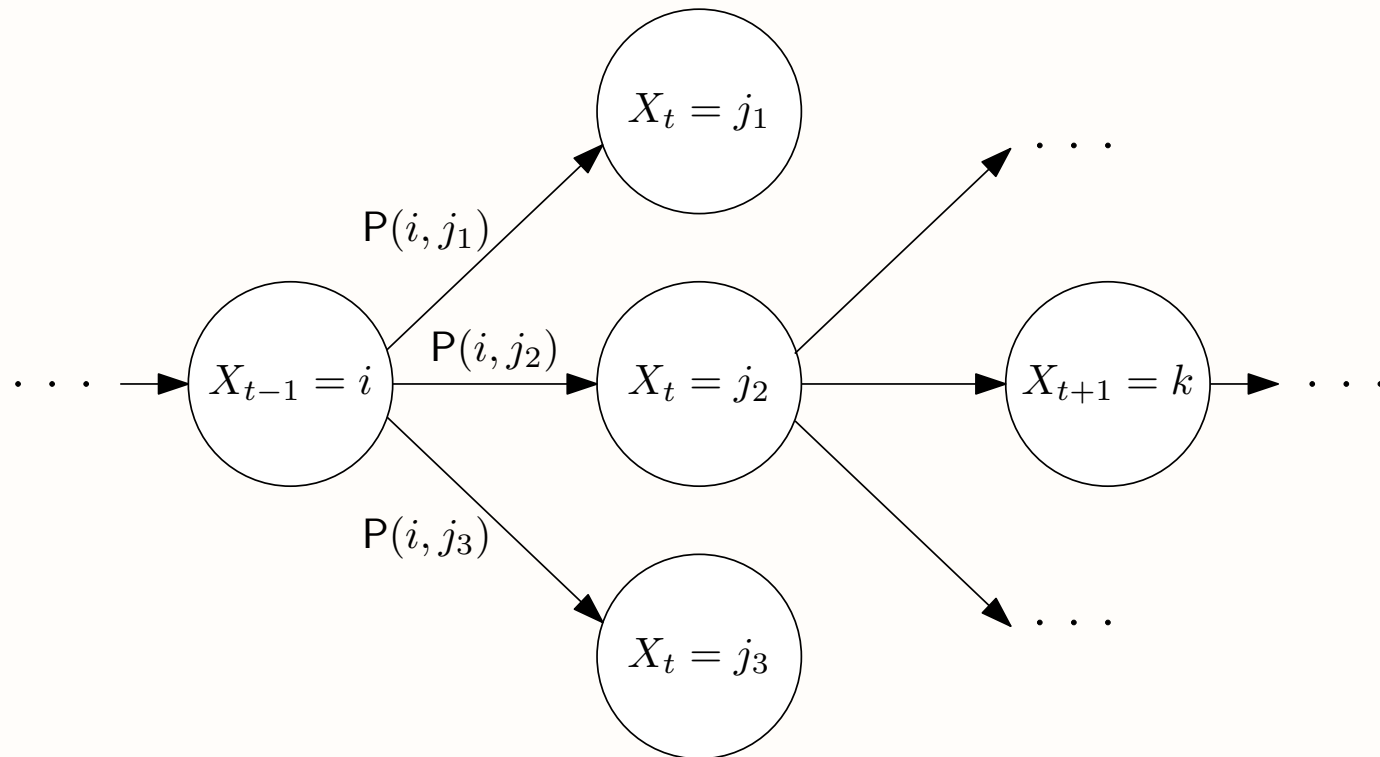


## Sequential decision making under uncertainty (2)



# Basic model: Markov chains

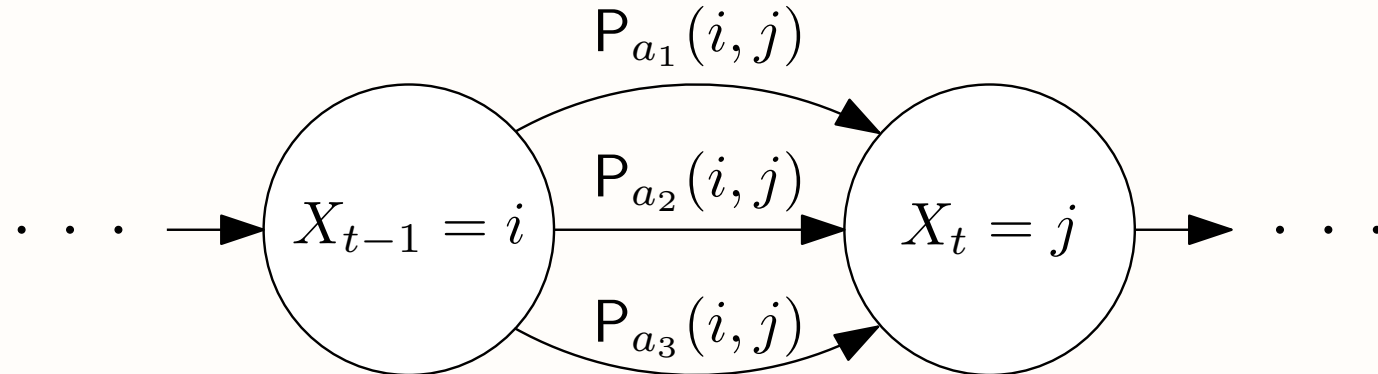
The basic model of *Markov chains* describes (first order) discrete-time dynamic systems.





## Adding control

In *controlled Markov chains*, the transition probabilities depend on a control parameter  $a$ .





# Markov decision processes

A *Markov decision process* (MDP) is a controlled Markov chain endowed with a performance criterion (Puterman, 1994; Bertsekas, 2000).

- The decision-maker receives a numerical reward  $R_t$  for each time instant  $t$ ;
- The decision-maker must optimize some long-run optimality criterion, e.g.,

$$J_{\text{av}} = \lim_{T \rightarrow \infty} \frac{1}{T} \mathbb{E} \left[ \sum_{t=1}^T R_t \right]; \quad J_{\text{disc}} = \mathbb{E} \left[ \sum_{t=1}^{\infty} \gamma^t R_t \right].$$





## Considering partial observability

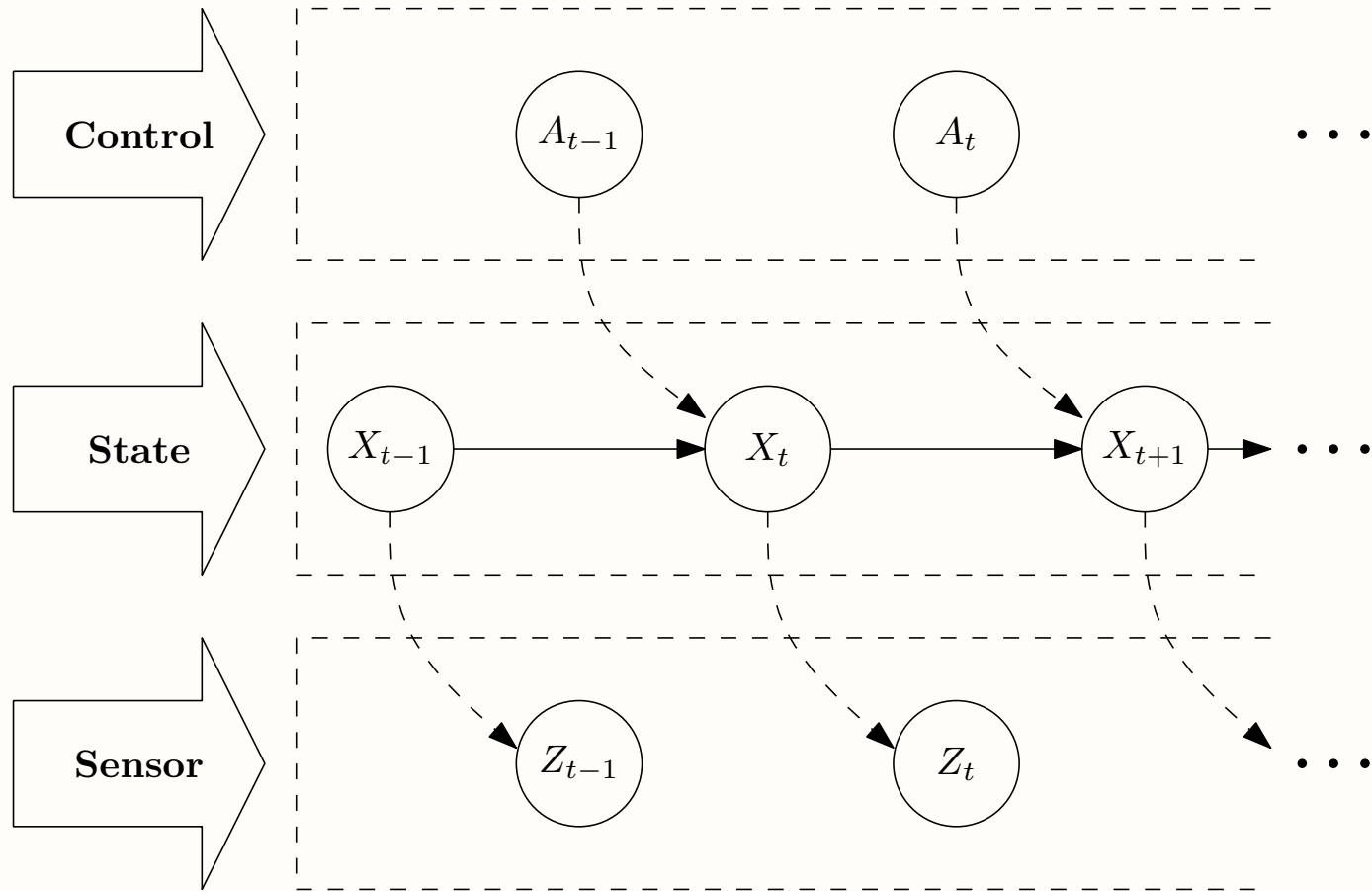
A *partially observable MDP* (POMDP) is an MDP where the decision maker is not able to access all information relevant to the decision-making process (Kaelbling et al., 1998).

- The decision-maker receives an observation  $Z_t$  for each time instant  $t$ ;
- The observation depends on the state of the underlying Markov chain;





# Considering partial observability (1)





## Multiple decision-makers

- *Stochastic games* (aka Markov games) provide a multi-agent generalization of MDPs (Shapley, 1953);
- In stochastic games, the control parameter depends on the choice of several *independent* decision-makers;
- In stochastic games, each decision-maker ( $k$ ) can receive a different reward  $R_t^k$  at each time instant  $t$ .





## Multiple decision-makers (1)

In stochastic games, as in MDPs,

- Each decision-maker ( $k$ ) must optimize its own long-run optimality criterion, e.g.,

$$J_{\text{av}}^k = \lim_{T \rightarrow \infty} \frac{1}{T} \mathbb{E} \left[ \sum_{t=1}^T R_t^k \right] ; \quad J_{\text{disc}}^k = \mathbb{E} \left[ \sum_{t=1}^{\infty} \gamma^t R_t^k \right] ;$$

- Partial state observability can be considered, leading to the framework of *partially observable stochastic games* (POSGs).





Fully observable:

- Multiagent MDPs (Boutilier, 1996).

Partially observable:

- Partially observable stochastic games (Hansen et al., 2004).
- Decentralized POMDPs (Bernstein et al., 2002).
- Interactive POMDPs (Gmytrasiewicz and Doshi, 2005).
- Each agent only observes its own observation.





## Model based

- Basic: dynamic programming (Bellman, 1957), value iteration, policy iteration.
- Advanced: prioritized sweeping, function approximators.

## Model free, reinforcement learning (Sutton and Barto, 1998)

- Basic: Q-learning, TD( $\lambda$ ), SARSA, actor-critic.
- Advanced: generalization in infinite state spaces, exploration/exploitation issues.







# Techniques for partially observable environments

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## Model based (POMDP)

- Exact methods (Monahan, 1982; Cheng, 1988; Cassandra et al., 1994; Zhang and Liu, 1996)
- Heuristic methods: based on MDP solution.
- Approximate methods: gradient descent, policy search, point-based techniques.

## Other topics

- Predictive State Representations (Littman et al., 2002).
- Reinforcement learning in POMDPs, PSRs.





## Model based:

- Hansen et al. (2004)'s dynamic programming.
- JESP (Nair et al., 2003).
- Bayesian game approximation (Emery-Montemerlo et al., 2004).

## Model free:

- Minimax-Q (Littman, 1994)
- FriendFoe-Q (Littman, 2001)
- Nash-Q, multi-agent DYNA-Q, correlated-Q.
- Learning coordination.





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# Reading group

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Questions to be answered:

- What topics shall we cover?
- When shall we meet? How often?
- Schedule, volunteers?





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